**Formulas for growth, time, convergence**

**Finding the rates of growth**

***Discrete***

$$g\_{d}=\left(\frac{X\_{i}}{X\_{0}}\right)^{\left(\frac{1}{n}\right)}-1$$

***Continuous***

$$g\_{c}=\frac{ln\left(\frac{X\_{i}}{X\_{0}}\right)}{n}=\frac{ln\left(X\_{i}\right)-ln\left(X\_{0}\right)}{n}$$

Where $g\_{d},g\_{c},X\_{i, }X\_{0,}n$ refer to rates of growth **d**iscrete and **c**ontinuous, the values of the variables being measured (GDP, GDP per capita, population, etc.) at the end and at the beginning of the period under analysis, and the number of years between the beginning and the end of such a period, respectively.

**Finding the time it takes to achieve a specified goal**

***Time it takes to reach a certain goal (the level of income, population, etc.) from a defined starting position at a given rate of growth***

$$t=\frac{ln\left(\frac{X\_{t}}{X\_{s}}\right)}{ln\left(1+r\right)}=\frac{ln\left(X\_{t}\right)-ln\left(X\_{s}\right)}{ln\left(1+r\right)}$$

Where $X\_{t},X\_{s}, r$ refer to the **t**arget and the **s**tarting levels of the variables concerned and the rate of growth, respectively.

Depends on the starting level, the desirable (goal) level and the rate of growth of the variables concerned (GDP, GDP per capita, population, etc.)

***Time it takes to increase the value of a variable (the level of income, population, etc.) by a certain defined amount, Z, given a certain rate of growth***

$$t=\frac{ln\left(Z\right)}{ln\left(1+r\right)}$$

Where Z and r refer to the amount by which the variable concerned should increase (2 for double, 3 for treble, and so on) and the rate of growth, respectively.

It’s independent of the value of the variables concerned, depending only on the rate of growth and by how much the variables concerned must grow. The numerator is the natural log of the number of times, Z, we wish the variables concerned to grow. For example, if Z = 2, (so, we wish to find the time it takes for the value of the variable concerned to double), then

$$t=\frac{ln\left(2\right)}{ln\left(1+r\right)}$$

**Convergence**

***Finding the time it takes for a poorer economy to achieve the current level of income per capita of a richer (wealthier) economy, given the rate of growth of the poorer***

$$t=\frac{ln\left(\frac{X\_{w}}{X\_{p}}\right)}{ln\left(1+r\right)\_{p}}=\frac{ln\left(X\right)\_{w}-ln\left(X\right)\_{p}}{ln\left(1+r\right)\_{p}}$$

Where subscripts w and p refer to wealthy and poor economies, respectively.

***Finding the time it takes for a poor economy to catch up with a richer economy, given their current levels of income and rates of growth*** *(note: the poorer economy will catch up with the richer if and only if the rate of growth of the poorer is higher than that of the richer)*

$$t=\frac{ln\left(\frac{X\_{w}}{X\_{p}}\right)}{ln\left(\frac{\left(1+r\right)\_{p}}{\left(1+r\right)\_{w}}\right)}=\frac{ln\left(X\right)\_{w}-ln\left(X\right)\_{p}}{ln\left(1+r\right)\_{p}-ln\left(1+r\right)\_{w}}$$